The expressional limits of formal languages in the notion of observation

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Abstract

In this article I deal with the notion of observation in the most fundamental sense and its formal representation by means of languages serving as expressional tools of formal-axiomatical theories. In doing so, I have taken this notion in two diverse contexts. In a first context as an epistemic notion linked to an elaboration of objects of a mathematical theory taken as registered facts of objective apprehension and then as notion linked to a process of measurement on the quantum-mechanical level. The second context in terms of which I deal with the notion of observation is that of phenomenological constitution basically as it is described in E. Husserl's texts on phenomenology of temporal consciousness. Taking that mathematical objects as formal-ontological objects in phenomenological constitution are based on perceptual objects by means of categorial intuition, the question is whether and under what theoretical assumptions we can, in principle, include quantum objects in the class of formal-ontological objects and thus inquire on the limits of their description in the language of a formal-axiomatical theory. On one hand, I derive an irreducibility on the level of observables as indecomposable atoms without any further syntactical content in formal representation and on the other a transcendence of a continuous substratum self-constituted as a kind of impredicative objective unity upon which is partly grounded the definition of an observational frame and the generation of a predicative universe of discourse.

Keywords. Continuum, flux of consciousness, individual, intentionality, observation, quantum measurement, quantum non-separability, transcendental.

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1 Introduction

I generally think that setting the limits of formal languages with respect to a notion of observation pretty much of the task consists in establishing an underlying all-encompassing theoretical background on which to be able to talk about observation either as an epistemic notion in the various contexts in which it is encountered or as a phenomenological notion linked to *a priori* defined acts of an 'observing' subject. My underlying interpretative scheme will be that of a phenomenological constitution of the intentional objects of experience by a 'participating' knowing subject in an object-like organization of the surrounding World-for us which from a certain viewpoint comes close to the version of Active Scientific Realism in Quantum Mechanics described e.g., in [18] and [19]. In my view this interpretation should lead to a 'representation' of well defined objects in the flux of consciousness by means of noetical-noematical constitution which involves a definition of objects as reidentifying bearers of predicates across phenomenological time. This has to do with a view of objects[1] in quantum mechanical context as general anticipative frameworks in a Boolean frame and then in a unifying mathematical - probabilistic tool corresponding to each experimental preparation.

Let me be a little bit more specific here about the meaning of the phenomenological terms of noetical and noematical described originally in E. Husserl's *Ideen I* ([12]). A noematical object manifests itself as a 'giveness' in the flux of a subject's consciousness and it is constituted by certain modes of being as such (e.g. with a proper to it predicative nest) i.e., as a well defined object immanent to the flux which can then be 'transformed' to a formal-ontological object and consequently a symbolic object of an analytical theory naturally including any formal mathematical theory. It can then be said to be given apodictically in experience: (1) it can be recognized by a perceiver directly as a manifested essence in any perceptual judgement (2) it can be predicated as existing according to the descriptive norms of a language and (3) it can be verified as such (a reidentifying object) in multiple acts more or less at will ([11]).

In contrast, an intentional object by hyletical-noetical apprehension (*Wahrnehmung*) can be only thought of as an aprioric orientation of intentionality by its sole virtue of

[1] If we regard as phenomenal objects those given 'primordially in perception' and represented in consciousness then interpretative objects like atoms, electrons, etc. can be also regarded as given by experience and thus considered as real to the extent that their 'reality' is based upon the interpretation of sensible signs in an experimental situation (see: [11], I, p. 5).

being given as such 'in person' in front of the consciousness inside the open horizon of the World-for us.[2] It is given within a horizon whose outer 'layer' is the boundary between the intentional object and the World-for us; the latter, is meant as not being this object or any of its parts and moreover it is the 'field' of all next possible noetical apprehensions. These intentional objects as most fundamental objects of intentionality cannot be reduced to a lower level of apprehension and this is seemingly the reason for which in their subsequent temporal constitution as individual noematical objects corresponding to 'state-of-things' they bear no 'inner' content at least not one describable by any analytical means.

My interpretational scheme will be also that of a transcendental reduction of a phenomenological type when it comes to the notion of constitution in-itself as an objectivated, homogeneous, continuous unity 'external' to its immanent objects on which to be able to constitute well defined objects of 'observation'.[3] This continuity in the sense of an underlying, impredicative and continuous substratum that makes possible to reinstate objects as bearers of predicates in kinematical interdependence in each experimental context is reduced to a transcendental subjectivity in the self-constitution of each one's flux of consciousness ([13], pp. 295-96). I also draw attention onto how the unity of a fulfilled time consciousness is reflected in the language of the mathematical theory of Quantum Mechanics in the form of classical continuity assumptions in the description of certain quantum phenomena or in the form of the state vector representing a quantum object immediately after measurement in a quantum experiment.

What can be derived from the mathematical description of quantum phenomena is the possibility to refer to entities as observational and then syntactical individuals in disentangled states and further the possibility to describe them by means of continuous transformations across time (e.g. state vectors or Dirac transformations) which implies, in turn, a continuity of the parameter time. I link these two fundamental possibilities in a phenomenologically motivated orientation to: (i) The existence of objects of intentionality within an outer and inner horizon of noetical apprehension - conditioned on the existence of a relation of intentional character[4] between a knowing subject and an object - which are then in noematical constitution bearers of a fundamental predicative formation and (ii) the impredicativity of the self-constituting unity of the time flux of consciousness leading finally to the transcendence of the pure ego in Husserlian terminology. I view these two fundamental irreducibilities in subsections 3.2 and 3.3 as playing an underlying role in the mathematical

[2]The World-for us or Life-World in Husserlian terminology can be roughly described to a non-phenomenologist as the physical world in its ever receding horizon including in intersubjective sense all knowing subjects in a special kind of presence in the World. More on this in E. Husserl's *The Crisis of European Sciences and Transcendental Phenomenology* ([15]).

[3]I note here that there exist certain approaches claiming a transcendental deduction of Quantum Mechanics (**QM**) such as M. Bitbol's ([2]) in the sense of a Kantian-type reduction of certain particularities of quantum description (e.g. superposition of states, continuity of the state vector) to their corresponding correlates in internal modes of functioning of human consciousness or J. Hintikka's ideas on a shift from a passive reception of objects to their effective research and instrumental shaping on the part of the knowing subject.

[4]This is not to be understood as a relation of some kind of psychological content. By intentional relation, which is a phenomenological term, it is meant something fundamentally deeper and aprioric. To a non-expert in Phenomenology it can be described in one phrase as grounding an aprioric necessity of the orientation of a subject's consciousness to the object of its orientation.

description of certain quantum phenomena such as the Bohm-Aharonov effect and quantum non-separability. An approach to quantum non-separability motivated to some extent by phenomenological concerns has been provided in [18] and [19].

Finally, regarding mathematical objects as not perceptual objects in literal sense yet founded on perception[5] I look, mainly in section 2, into how the aforementioned irreducibilities are formalized in the language of axiomatical mathematical theories, notably in *Zermelo-Fraenkel* with *Axiom of Choice* Theory (**ZFC**), and consequently in the mathematical metatheory of **QM**. They set, in effect, as I shall try to show, the expressional limits of a formal language in the notion of observation because they stand as irreducibilities of a rather phenomenological and certainly of a non-analytical character. By this measure, they can be considered as a unifying substratum of both quantum-mechanical observation (in the sense of a fundamental character 'observation') and the logical-axiomatical structure of the corresponding formal theory.

From this aspect, if we adhere to the view that mathematical Continuum is primarily based on the intuitive Continuum of our real experience and it can be modelized after the self-constituted Continuum of the time flux of consciousness (which was roughly both L.E.J. Brouwer's and H. Weyl's view of the matter, see: [27]), we plausibly expect the transcendental root of phenomenological Continuum to be somehow reflected in the axiomatization of mathematical theories pertaining to a notion of mathematical Continuum. This is what seems to happen with the independence from the rest of the axioms of **ZF-C** of statements making claims about fundamental properties or the cardinality of the Continuum (e.g. *Continuum Hypothesis*, the *Axiom of Choice*, *Suslin's Hypothesis*, etc). I simply add as indirectly relative to this, Gödel's view that the mathematical essences we intuit could not be linguistic conventions in the sense that: "*instead of clarifying the meaning of abstract and non-finitary mathematical concepts by explaining them in terms of syntactical rules, abstract and non-finitary concepts are used to formulate the syntactical rules*" ([26], p. 193).

2 Noematical objects as individuals of mathematical theories

It seems to me purposeful to draw attention to what is most fundamental, in fact what is irreducible in the build-up of analytical statements of any degree of complexity. If we assume that any analytical statement incorporates noematical objects[6] in the sense of signification objects (*Sinnesobjekte*) acting as predicate bearers (*Seinssobjekte*) in noematical constitution together with any doxical modalities reflecting consciousness-based states e.g. doubt, certainty, negation, etc., then any attempt at a radical 'deconstruction' of the analytical structure of any sentence would inevitably reduce to statements about individuals and

[5] In E. Husserl's view, perception by virtue of perceptual acts provides the concrete, immediate and non-reflective basis for all our experiences and thus provides also the basis for any intuition of abstract objects. For details, see R. Tieszen's [25], pp. 412-15.

[6] This kind of noematical objects, e.g. syntactical individuals of categorial formulas, numerical symbols, functions of Pure Analysis, Euclidean or non-Euclidean domains of such functions etc. were characterized by Husserl as referring to 'state-of-things' (*Sachverhalte*) which are intentionalities towards an 'empty something' (*Leeretwas*) ([12], p. 33)

to properties correlated with their very nature as individuals. As E. Husserl claimed, those sentences are no more of an analytical nature but of a rather phenomenological one leading to a kind of 'observation' of intentional character. This radical reduction which in addition to predicate bearers as such reaches their predicative environment as well, should fundamentally attach at least the $\in$ predicate as a noematical correlate to each such predicate bearer in case we talk about syntactical atoms of analytical - mathematical formulas.

This type of reduction is meant as the result of the elimination of all possible doxical modalities in the construction of analytical sentences of any level of complexity as, for instance, in general statements expressing doubt (S should be p), corroboration (S is in fact p) or negation (S is not p) and so on, including also forms uniquely defined by their syntactical structure e.g. where one quantifies over elements satisfying a particular analytical property S ($\forall p\ S$ or $\exists p\ S$). What is left ultimately is a multiplicity of individuals (or a collection of such individuals by phenomenological association) as intentionally perceived and constituted as reidentifying noematical objects in a varying predicate environment with $\in$ predicate intentionally attached to them as an irreducible, non-logical notion of order. The predicative nests intentionally attached to individuals (in the sense of syntactical atoms) were termed by Husserl as *Kernformen* in [13] and were supposed to invariably define the essential nature of individuals - substrates in the subsequent construction of analytical statements of a higher order. From my point of view the $\in$ predicate is even more fundamental than the equality predicate at least in analytical representation for any notion of equality presupposes a notion of mutual inclusion. This is reflected in a fundamental way by the adoption of *Extensionality Axiom*:

$$\forall X\ \forall Y(\forall u(u \in X \longleftrightarrow u \in Y) \longrightarrow X = Y)$$

in the foundations of standard set theory as well as in almost any formal theory treating sets as definite collections of objects.

My claim is that this radical reduction to individuals - predicate bearers as ultimate phenomenological substrates of analytical statements provides a satisfactory interpretational framework for the role of urelements of transitive classes under $\in$ predication. As it will become more clear in next section, a common foundation to both formal-ontological objects of categorial structures and to objects of quantum 'observation' in their noematical representation in consciousness lies in the notion of intentional objects constituted in an all-inclusive objectivity of consciousness as reidentifying across time predicate bearers.

Any intentional object in the possibly lowest level of 'observation' can, in Husserl's view, be only an individual - substrate deprived of any inner structure (even lacking a temporal form), at least not one expressible by any analytical means together with an *a priori* predicative formation by virtue of which it can be perceived as a unique noematical object appropriating *eo ipso* a relational property with respect to any other such object. It is by all accounts this essential characteristic together with the retentional[7] character of

[7]The terms retention and protention are purely phenomenological terms and can be roughly communicated to a non-expert in Phenomenology respectively as a kind of immediate conservation in memory (retention) and immediate expectation of original impression (protention); they are described to be of an aprioric and not of psychological character in the constitution of the sequence of original impressions in the flux of consciousness. For more details, see [14].

the constitution process that it is possible to re-identify any intentional object of primordial experience as the one and same noematical object x under varying predicative situations. Any attempt to pass from those individuals perceived as a self-donated presence in front of the intentionality of consciousness to a constituted objectivity of a higher order can only entail circularities in description or *a priori* terms. For instance, in *Ideen I* Husserl referred to the multi-ray intentionality of the synthetic consciousness which turns by an essentially *a priori* mode the apprehension of a collection of objectivities into an apprehension of a single objective whole by what he termed a monothetical act whose *wesensmässig* (by essence) mode evidently points to a creeping transcendence ([12], p. 276).

I'll comment on phenomenological transcendence which was described *in extenso* in terms of the self-constitution of time consciousness by Husserl in [14] later in Sec. 4; now I call attention to irreducible individuals as formal - ontological objects with their inherent predicative formation and to their implicit role in determining properties of transitive classes concerning, in particular, the proof of absoluteness of certain categorial formulas.[8]

It is very important, for instance, to assure absoluteness of certain bounded quantifier set-theoretical formulas in the build-up of hierarchies of transitive classes L_α in Gödel's Constructible Universe L to prove that it serves as a model of **ZFC** plus **CH** [**ZF** theory + **AC** (*Axiom of Choice*) + **CH** (*Continuum Hypothesis*)]. For such formulas the property of absoluteness basically is related with the transitivity property of the corresponding class; in intuitive terms it has much to do with the invariability of the $\in$ - predicative character of the zero-level elements of the original transitive model in the recursive definition of classes of any order inside it.[9] For instance, by transitivity property any bounded quantifier formula φ of the form

$$(\exists u \in x)\ \psi \ \text{ or } \ (\forall u \in x)\ \psi$$

is absolute between any transitive models M and N whenever formula ψ is.

The simple proof is based on two assumptions. First, that in the inductive definition of absolute formulas any atomic formula ψ of the form $i \in j$ and $i = j$ is absolute and second, that any bounded variable u of the formula φ is the 'reflection' of a certain invariably the same urelement u_i under $\in$ - predication in M.[10]

Both assumptions reduce to admitting the possibility of existence of irreducible individuals retaining invariably their double nature as individuals-as such and as members of the (transitive) class to which they belong. Whether they should be objects of stratified categorial formulas in a logical - mathematical statement or well defined objects of quantum observation expressible as formal - ontological objects in the syntactical norms of a

[8] In rough terms an absolute formula φ inside a model X can be described as keeping the 'mirror' image of itself in any other model Y ordered by set inclusion ($X \subset Y$) with respect to the original model X.

[9] A class M is transitive if for any $x, y, z \in M$ whenever $x \in y$ and $y \in z$, $x \in z$. This is equivalent to the statement that whenever $x \in M$ and x is not a zero-level element (i.e., an urelement under $\in$ predicate) then $x \subseteq M$.

[10] It is an immediate consequence of the transitivity property of model M that it satisfies the Axiom of Extensionality by absoluteness of the bounded quantifier formulas

$$\forall X\ \forall Y\ [(\forall u \in X) \to (u \in Y) \wedge (\forall u \in Y) \to (u \in X)] \longrightarrow X = Y]$$

([17], pp. 82-83).

formal - analytical discourse changes nothing as to the essence of their individuality and their aprioric predication thus leading to a view of them as 'transformations' of intentional objects of noetical apprehension. The latter case (i.e. quantum objects) will be discussed in more detail in the next subsection.

Whether we may introduce individuals as urelements dropping the Axiom of Extensionality as Fraenkel and Mostowski did in constructing appropriate models in which the *Axiom of Choice* fails[11] or dismiss them altogether reserving this denomination only for null-set ($\emptyset$) yet retaining a notion of individuals as first-level elements in a cumulative type structure, the underlying idea of 'indecomposable' individuals preserving invariably their syntactical and (in appropriate interpretation) semantical content remains fundamentally the same.

Even viewing urelements of an extended Zermelo-Fraenkel universe (ZFU, $\in$) as not identical yet indistinguishable elements by the definition of $\mathcal{A}$ - indistinguishability inside a relational structure $\mathcal{A} = \langle D, \{R_i\}_{i \in I} \rangle$ (as proposed by Krause and Coelho in [20]) they can be easily made distinguishable by associating to any collection of them an ordinal number making thus possible to talk about a collection $\sigma_0,\ \sigma_1,\ \sigma_3, ...,\ \sigma_{n-1}$ of such objects. This is a result of the simple proof that any ordinal as a well-ordered structure $\langle A,\ < \rangle$ is a rigid structure, i.e. the only automorphism in this structure is the identity function ([20], p. 201). In other words, in a rigid structure $\mathcal{A}$ the notion of not identical elements and that of $\mathcal{A}$ - distinguishable elements coincide. Let us note that the question of the individuality of entities in the context of quantum mechanics has provided for much theoretical discussion on the nature of quantum objects as they are regarded by some physicists (notably by Scrödinger) as non-individuals upon which a notion of identity does not make sense or by others as bearing a kind of intrinsic individuality by which though they might be "qualitatively the same in all aspects representable in quantum mechanical models yet numerically distinct" ([28], p. 376).[12]

Individuals as purely *qua* individuals whether we refer to an axiomatical-mathematical model or to a mathematical modeling of quantum mechanics are to be thought on a formal level as reflections of irreducible intentionalities of consciousness of an *a priori* character and they should not be necessarily identified on a formal-ontological level with certain elements of standard or non-standard theories at least not in the absence of the *Foundation Axiom* of **ZFC** theory. They can represent any entity in the structure of an appropriate formal language as long as it is taken as elementary and not further reducible in the sense we have described.[13]

[11] P. Cohen dismissed those urelements as fictitious objects x_i such that $\forall y\ (\neg y \in x_i)$ yet $x_i \neq x_j$ for $i \neq j$ ([20], p. 202). Yet in the sense of noematical individuals inside the unity of consciousness that I have proposed it makes sense to talk about such objects.

[12] In [20] a model-theoretical characterization is proposed of the two opposing views on the question of the individuality of quantum particles in terms of a trivial and a non-trivial rigid expansion of a relational structure $\mathcal{A} = \langle D, \{R_i\}_{i \in I} \rangle$. It is evident by the arguments employed in this article that its authors have the view that the mathematical structure of Quantum Mechanics has a non-trivial rigid expansion (i.e. not one by trivially adjoining the ordinal structure) whose physical intuition is that quantum entities are somehow 'intrinsically' distinguishable one from another.

[13] In [3], C. Brink and I. Rewitzky derive by a proper mathematical modelisation involving Priestley duality (something close to Stone duality) that it is not essentially different whether we talk either of individuals (things), properties, or facts in the world, establishing, in effect, an intranslatability between an ontology of individuals (nominalism), an ontology of properties (realism) and an ontology of facts (factualism).

Of course many other relations or properties other than the $\in$ predicate can be predicated to any individual as an object of a formal-ontological discourse but what we are most interested in here is to reach the most fundamental, the not further reducible level of predication. Evidently, this kind of irreducibility connects with a notion of ordinals as a transitive and well-ordered structure within a mathematical - axiomatical system. Taking into account that by definition an ordinal number is a transitive and well-ordered by $\in$ inclusion set it seems natural to conjecture that transitive models of **ZFC** should be determined by their sets of ordinals. In fact, this was proved by P. Vopěnka and B. Balcar in [29] where any transitive models M, N of **ZFC** are proved to be equal ($M = N$) whenever M and N have the same sets of ordinals with the restriction that AC is satisfied in M ($M \models AC$).

It is not without importance here that **AC** should be satisfied at least in model M and it gives the motivation to a brief review of this independent infinity axiom within the scope of the present work. The intuition of the *Axiom of Choice* is that we can, in principle, apply a criterion of choice at any infinity level which would provide us with the possibility to select an irreducible individual (think of it as an urelement of a formal theory) together with an inherent $\in$ relation attached to it, among any other potential choices or in phenomenological terms among any other possible intentional 'observations'.

In a phenomenological approach, I argue that a notion of ordering may be automatically induced by any object of intentionality at the level of hyletical - noetical apprehension by the sole virtue of the intentional 'property' of the object in question to bear an outer horizon, i.e. that part of the Life-World that is not the object or parts of the object. Evidently, this 'property' provides for a complementary domain of 'observation' for a next potential noetical apprehension which by its very enactment provides for a new complementary domain and so on. This way a notion of well-ordering may be grounded on the noetical level of intentionality with regard to a transformation thereafter of an hyletical-noetical object to a noematical one possibly belonging to an aggregation of other such objects in the continuous unity of the absolute flux of consciousness. Now, what is left after discarding all other details of constitution is the possibility to 'observe' (and retain) intentionally individuals-as such as protentions of intentionality in the domain of 'observation'. By this deconstruction process, a fundamental reduction of the *Axiom of Choice* as a series of intentional acts of an *a priori* character inducing in posterior sense a well-ordering among any aggregation of formal-ontological objects seems to me plausible.

3 Can quantum mechanical interpretation be related to a phenomenology of constitution?

3.1 Some remarks on the mathematical language of Quantum Mechanics

In this section I shall argue for the possibility of an interpretation of Quantum Mechanics along phenomenological lines especially in connection with the notion of the intentional relation subject-object and the constitution of noematical objects as well defined objects in the self-constituted unity of consciousness. This is an approach that to my knowledge is pretty much new though there have already been various interpretations beyond the main-

stream options of realism and instrumentalism, such as M. Bitbol's views in [2] motivated by an attempt at a transcendental deduction of Quantum Mechanics or the *Many-Worlds Interpretation* of Quantum Mechanics (**MWI**) taken as B. De Witt's interpretation of H. Everett's 'relative state' formulation of **QM**.[14]

In M. Bitbol's approach I retain, first, his view of an original type transcendental deduction summed up as providing an internal correlation between a unifying mode of appearances of phenomena and certain laws of understanding considered as preconditions of experience. This seems, in a quite general form, to shift the view towards a field where the mental faculties of a subject might actively take part in grounding experience as such and also in shaping up the objects of experience. And, second, I think we should keep in mind the meaning of his constraint of contextualization which corresponds a Boolean subframe to each experimental preparation linked to a unified mathematical tool of probabilistic prediction irrespective of the context associated to the measurement that follows the preparation ([2], p. 11). In such a case, based on the notion of a reidentifying object across time (which in my view presupposes the existence of an otherwise irreducible intentional relation subject/object) we can ascribe to each experimental preparation a unified, predictive (non-Kolmogorovian) tool whose valuations are associated with a Boolean framework irrespective of the context associated to the measurement that follows the preparation.

This seems interesting to the extent that: a) it presupposes an object-like organization of phenomena to be described in the language of a Boolean observer 'attached' to each experimental situation and b) it implies a unifying mathematical tool bridging, in effect, the contextual frames of the preparation of an experiment and its measurement.

The former condition can be understood as leading to the following assumptions. The indirect introduction of a participating 'observer' who has a particular mode of 'observation', then a particular mode of constituting his observations and reproducing them in a predicable object-like universe and a particular language to describe them as well defined objects. In the case of an experimental preparation and measurement this is inherently linked with an observer's capacity to transform his intentionalities in terms of noetical-hyletical apprehensions of the real world to noematical objects in terms of reidentifying objects across time and thus well defined bearers of predicates in an ordered context. Then he would be able to talk, in principle, about the ontological nature of these constituted objects irrespectively of whether they are considered as objects under predication of a formal or of a common natural language. For they have become objects of a formal ontology which in reverse order reduce by phenomenological intentionality to individuals- as such bearers of an outer horizon in hyletical - noetical apprehension.

In view of the aforesaid I propose an interpretation (in terms of a noetical - noematical constitution) of the presence of a knowing subject that performs quantum measurements via a measuring apparatus in the following fashion: The measured property produces a macroscopic effect on the instrument (e.g. a pointer reading or a track in a bubble chamber) which is a material sign. This can be considered as having a double reality; its material one

[14] In connection with this consciousness-related orientation, I specifically refer to its 'psychological' version where the quantum measurement process is roughly reduced to a 'splitting' of a single consciousness before interaction to several afterwards yet retaining by some psychological mode its unity through time. See, [6] and [7].

as a pointer sign or a bubble track and an intentional reality proper to it as a sign susceptible to be constituted at a next stage as a formal-ontological object. A sign regardless of its particular material content has the mode of being a sign-as such and in being so it can be thought of as an intentional object of noetical apprehension by virtue of being merely a so-called 'state-of-things' (*Sachverhalt*), in other words an 'empty something'[15]; it should then be apriorically directed to a knowing subject performing the experiment by means of a measuring device. In any case, it may be assumed that the signs of a measuring apparatus are symbols of certain physical properties (natural symbols) insofar as they are uniquely determined by the interaction with a quantum entity in terms of which they 'translate' the hidden state of the quantum entity into uniquely determined sensible signs ([11], p. 174-175). Evidently, these signs which are part of the 'physicalistic language' of the measuring apparatus can be considered as intentional objects for the performing subject who can then turn them to linguistic symbols; the latter is conditioned on his capacity to constitute them as well defined noematical objects in his flux of consciousness. As symbols of a linguistic statement they are not just material reality signs but they are parts of a predicative environment by being symbols as-such corresponding to 'state-of-things' which are moreover bearers of two important properties: 1) They do not determine a unique linguistic statement. By being symbols-as such they can be arguments of equivalent logical-mathematical formulas inasmuch as they are abstractions of unique material signs and 2) They are devoid of any inner analytical content as they are linguistic symbols abstracting in each case a unique and irreducible intentional object, e.g. the sign-as such of the bubble track of a particle.

At the stage a knowing subject will be able to represent noematical objects as such and such well defined objects of discourse in such or other internal noematical mode he should by necessity have constituted them already in a kind of synthetic unity to be able to talk about them together at once; and this unity should undeniably be a temporal unity. But arguably this reduces to a constitution of internal time in the form of a continuous unity of time consciousness which has almost nothing to do with external (or scientific) time e.g. the time frame of classical or relativistic theory.

In fact, both quantization conditions and the continuous wave-like propagation of phenomena, stemming from the formalism of Quantum Mechanics with appropriate boundary conditions, are due to the intrinsic property of quantum objects to be 'embeddable' as outcomes of sufficiently reproducible experiments to a unified meta-contextual frame of probabilistic description. This possibility involves, on a phenomenological level, at once both a relationship subject-object of an intentional character and the noematical constitution of intentional objects as well defined objects in the unity of consciousness; in a quantum experiment this is translatable to an embedding of reproducible 'observations' in a meta-contextual Boolean subframe.[16] In addition and independently of the context that

[15]By the transformation of originally given intentional objects as 'state-of-things' to formal-ontological objects it is possible for a physical interrelation to be formalized mathematically with numerical (or mathematical) symbols corresponding to observable signs furnished by the intermediary of a measuring instrument.

[16]My intentional/constitutional approach is not directly connected with the general stance one might have on the question of the nature of quantum particles as there is no unconditioned description of them. Anyway I note that in the light of EPR critique and the antinomies of entanglement states Schrödinger

follows the preparation of a quantum experiment there should be some intrinsic way by which quantum objects as intentional ones become reidentifying objects of a constituting consciousness invariably over (internal) temporal unity. Eliminating then all time-related modes of noematical constitution (e.g., simultaneity, succession, casual relationship) there should be a temporal substratum of the predicable universe of Quantum Mechanics whose temporality should be something radically different than the ordinary objective time of a classical macroscopic system. This may lead to argue that the unifying, meta-contextual time of the predictive tool should be the objectivated 'reflection' of the absolute flux of time consciousness whose objectivation cannot be described but as a sort of 'mirror' reflection of its ever *in-act* self.

I close this subsection by referring to a well-known quantum effect where the derivation of quantum conditions by classical continuity assumptions as constraints provides a clue to the necessity of assumption of an impredicative temporal substratum to which I just referred above.

In the case of the potential well, for instance, the discrete eigenvalues of the energy operator is the formal result of continuity assumptions about the wave function ψ on the boundaries of the potential well and of constraints put on the wave function out of the classically permitted region, in the 'observational limit' to infinity.[17]

Both constraints underlie an observational capacity linked at least indirectly to a subjective notion of temporal continuum. In that sense, the condition of continuity at $x = \pm\frac{a}{2}$ of the wave function and its derivative implies the underlying existence of a continuous substratum of internal time providing a continuous domain for the particle state function whereas normalization condition $\lim_{x \longrightarrow \pm\infty} \Psi(x) = 0$ which is in accord with the classical intuition that the formal representation of the physical state of a free particle should approach zero at infinite limit, is a classical limit equation presupposing a continuous space-time substructure. It seems that a continuous substratum of time-consciousness which is also asserted (the primordial intuition of mathematics) in a non-standard notion of mathematical Continuum[18] should be implicitly assumed in setting classical limit equations $\lim_{x \longrightarrow \pm\infty} \Psi(x) = 0$

attempted a reinterpretation of the epistemological questions of quantum theory and a reexamination of the question of the individuality of particles. In fact, it was his attempt to interpret the formal results of the Bose-Einstein statistics which implicated an indiscernibility of monoatomic gas molecules that led him to abandon the particle interpretation and adopt the undulatory view in terms of which the gas as a physical system should be considered as a system of stationary waves in which molecules are just states of excitation energy deprived in this way of individuality. Later he gave a physical interpretation in electromagnetic sense to the wave function ψ as solution of his general equation but that was again of a wave configuration consisting in the superposition of all kinematically possible point-configurations of the system each one intervening by its special "weight" in the physically interpreted formula $\int \psi\psi^*$ (current density). Moreover his wave image proved more satisfactory in representing atomic transitions by an energy exchange between different vibrations rather than by a quantum leap between states in which case one cannot possibly describe the transition in time and space.

[17] Based on the continuity of the wave function of a free particle at the boundaries $x = \pm\frac{a}{2}$ of a potential well V we get the equations $\psi(\frac{a}{2}) = \psi(-\frac{a}{2})$ and $\psi'(\frac{a}{2}) = \psi'(-\frac{a}{2})$. For $x < -\frac{a}{2}$ or $x > \frac{a}{2}$, in the limit at infinity it must hold that $\lim_{x \longrightarrow \pm\infty} \Psi(x) = 0$ for the wave function Ψ of a free particle of energy E (with $E < V$) which in this region of the plane takes the form of a descending exponential function $\psi(x) = C\exp(k_1 x) + D\exp(-k_1 x)$ (see [22] or any other basic **QM** textbook).

[18] For a brief survey of L.E.J. Brouwer's and H. Weyl's modelization of mathematical Continuum after the

and equations of continuity of the wave function Ψ at the boundaries $x = \pm\frac{a}{2}$.

To sum up, discrete eigenvalues of bound states of a quantum system are in mathematical formulation partly an indirect consequence of classical limit and continuity assumptions against an underlying substratum of impredicative continuum of propagation. An objectivated, time-fulfilled Continuum (*erfüllte Kontinuum*) must be also presupposed in the phase of second (or field) quantization in the relativistic version of Quantum Mechanics where single particle wave functions of classical version are transformed into quantum field operators on quantum states defined on any space-time point. This is basically implemented by an extension of Langrangian formalism to field equations.

To demonstrate the underlying presence of this kind of impredicative continuous spatiotemporal substratum in the mathematical theory of quantum mechanics I refer, in the following subsection, to the well-known Bohm-Aharonov effect.

3.2 The case of the Bohm-Aharonov effect

The main purpose of my reference to the Bohm-Aharonov effect is to show that certain irregularities on the observational - physical level are reduced in mathematical tool to special topological properties of the relevant configuration space. In the specific effect the irregularity has to do with the presence of a solenoid causing a shift in the interference pattern of a double slit in the notable absence of an external magnetic field. Moreover, the physical effect observed which is the change in phase difference in the electron interference pattern $\Delta\delta = \frac{e}{\hbar}\int \mathrm{curl}A\ dS$ depends only on curlA[19] so that it could be deduced that an electron is influenced by fields which are only non-zero in regions inaccessible to it. In formal terms, this amounts to a non-locality of the integral $\oint Adr$. In short we could say that the Bohm-Aharonov effect owes to the non-trivial topology of the vacuum (in this particular case the space outside the solenoid) and the fact that electrodynamics is a gauge theory ([24], p. 101).

Being a bit more specific, without intending to enter into the details of the experimental context, the existence of the Bohm-Aharonov effect is essentially translatable to a topological situation where the configuration space of the null field is a plane with a hole in it, that is the non-simply connected circle S^1. In further mathematical elaboration, this generates a many-valued gauge function x mapping the group space S^1 onto the configuration space of the experiment $S^1 \times R$ such that not all such x are deformable to a constant gauge function ($x = \mathrm{const}$). In that case, it would produce $A_\mu = 0$ and no Bohm-Aharonov effect ([24], p. 105). In mathematical formalism the function x such that $A = \nabla x$ turns out to be a many-valued function and this becomes possible since the space in which it is defined is not simply connected. That is, the group space of the gauge group of electromagnetism $U(1)$ is the non-simply connected circle S^1 where, roughly speaking, a non-simply connected space is one in which not all curves may be continuously shrunk to a point.

If x were single-valued, then $B = \mathrm{curl}A = \mathrm{curl}\nabla x \equiv 0$ everywhere, so there would be no magnetic flux at all and consequently no physical effect taking into account that $\Delta\delta = \frac{e}{\hbar}\Phi$.

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phenomenological Continuum I refer to M. Van Atten's *et al* work in [27].

[19]The magnitude A is the vector potential which in classical physics, as it is well-known, is linked to the magnetic induction B by the formula $B = \mathrm{curl}A$.

In view of our previous discussion, we note in this specific quantum mechanical experiment a 'transformation' of the irregular observational characteristics of the quantum phenomenon into peculiarities in the topological texture of a spatiotemporal continuous substratum; in the particular case the peculiarity lies in the property of non-simply connected of the configuration space of the experimental context. But, in generating topological properties leading to certain discontinuities in configuration space one must assume, prior to the assumption of discontinuity gaps in topological structure, the constancy of an underlying spatiotemporal continuum across time which can in turn reduce to the constancy of a fulfilled time-consciousness self-constituted as a continuous unity 'bridging', in effect, the context of an experimental preparation with that of measurement.

3.3 Interpreting quantum non-separability

In quantum mechanical theory quantum non-separability arises first as a result of the principle of superposition of states and second from the impossibility to provide, given a compound system S and its corresponding Hilbert space H, a decomposition of it into a tensor product $H = H_1 \otimes H_2 \otimes ... \otimes H_N$ of the subsystem spaces H_i such that an observable A of S can be expressed in the canonical form $A = A_1 \otimes A_2 \otimes ... \otimes A_N$ of suitable observables of the subsystems S_i. Formally this is a result of the particular feature of the tensor product that it is not a restriction of the topological product $H = H_1 \times H_2 \times ... \times H_N$ but includes it as a proper subset. Given that in quantum mechanical theory there are no reasonable criteria that would guarantee the existence (and uniqueness) of such a tensor product decomposition of the whole system the question is how we could possibly derive it and on what terms on an operational level.[20]

V. Karakostas, for instance, discusses in [18] and [19] the question of non-separability from the point of view of Active Scientific Realism as presupposing the feasibility of the kinematical independence between a component subsystem of interest and an appropriate measuring system including its environment; it presupposes, in general, the separation between the observer and the observed. Taking the physical world as an unbroken whole we have to separate it, to perform a breakdown of the entanglement of subsystems. In what is called a Heisenberg cut, we have to decompose the compound entangled system into interacting but disentangled components that is, into measured objects on the one hand and measuring systems (uncorrelated observers in a broad sense) on the other with no (or insignificantly so) holistic correlations among them. By means of the Heisenberg cut can be generated well-defined separate objects in their contextual environments described in terms of a process of projecting the holistic non-Boolean domain of entangled quantum correlations into a meta-contextual Boolean frame that breaks the wholeness of nature by means of an

[20] A prototype of an EPR - correlated system experimentally confirmed is the compound system S of spin-singlet pairs. It consists of a pair (S_1, S_2) of spin $\frac{1}{2}$ particles in the singlet state

$$W = \frac{1}{\sqrt{2}} \{ \mid \psi_+ > \otimes \mid \phi_- > - \mid \psi_- > \otimes \mid \phi_+ > \},$$

where $\{ \mid \psi_\pm > \}$ and $\{ \mid \phi_\pm > \}$ are orthonormal bases of the two dimensional Hilbert spaces H_1 and H_2 associated with S_1 and S_2 respectively. In such a situation, it is theoretically predicted and experimentally confirmed that the spin components of S_1 and S_2 have always opposite spin orientations.

effective participancy in the physical world of a knowing/intentional subject ([18], pp. 300, 303). In fact, the notion of an effective participancy of a knowing/intentional subject in the physical world seems to imply the Aristotelian idea of *potentia* since, on a quantum level, for any effective observer inside the Life-World there should be two categories of entities, those posterior to his knowing/intentional acts which as already pointed out he has some inherent mode to recognize as well-defined objects and those prior to his purely intentional acts which should by necessity be for him mere potentialities; in that sense, "a quantum object exists, independently of any operational procedures, only in the sense of 'potentiality', namely, as being characterized by a set of potentially possible values for its various physical quantities that are actualized when the object is interacting with its environment or a pertinent experimental context" ([19], p. 290).

In view of description of the relation between a knowing subject and an object of his intentionality (in terms of noetical-noematical constitution), offered mainly in subsection 3.1, we may argue that there exists a certain convergence of the interpretational content of phenomenological analysis with the positions of Active Scientific Realism[21] inasmuch as:

The implementation of the Heisenberg cut can be taken in a fundamental sense as presupposing a notion of co-existence and also an idea of separation in a domain of intentional 'observation' between a consciousness intentionally directed to its object and the object initself as a direct and unambiguous presence in front of the intentionality of consciousness. By applying his intentionality a knowing subject creates a particular context to inquire e.g. on the 'hidden status' of an entangled quantum state in the following two stages: 1) on the noetical level by apprehending a sensible sign (of the measuring apparatus) as such in the sense that it could not be otherwise but apprehend it as a sign distinguishable from any other possible sign in the protention of his intentionality (cf: [16], p. 8); at this stage he has already lost his claim on an access to the inner reality of the entangled state for he noetically apprehends what he apprehends in the 'physicalistic language' of the apparatus and 2) thereafter he constitutes it as a noematical object immanent to his consciousness in the modes already described.

Moreover, I feel that the introduction of the effective presence of a knowing/intentional subject in the Life-World puts on a close footing these two interpretations with respect to the aristotelian notion of potentiality as they seem to somehow weaken the vaguely metaphysical character of this principle exactly by the introduction of an intentional/constituting subject as part of the Life-world. So, from a phenomenological point of view, a World in which pre-predicative structures (i.e. intentionality structures) linked to the presence of an intentional/constituting subject determine by 'anticipation' actual predicated instances may be defined as a domain of real possibility anterior to actuality. This seems partly to eliminate the vague ontological status - not to say purely metaphysical - of Aristotelian potentialities for it substitutes for the notion of a first *entelechy*, reached necessarily by regression ad infinitum of all classes of potentialities, the notion of at least one constituting subject in a pre-phenomenological World. Connected to the view above, is the assertion

[21] In a certain sense this approach is related to H. Everett's 'Relative State Interpretation' of Quantum Theory e.g. by means of a decoupling of world components $\psi^{(R)}$, $\psi^{(L)}$ of a certain superposition state $\psi(t) = e^{iHt}\phi(\varphi_R \pm \varphi_L)$ corresponding to a localization of consciousness not only in space and time but also in certain Hilbert space components (see example: [30], pp. 73-74).

that the inherently probabilistic nature of Quantum Mechanics may be interpreted as due to the irretrievable loss of information caused by the cut of a quantum non-separable whole in the measuring process. This means that in view of the reduction to the intentionalities of a subject performing a quantum experiment certain potentialities of a quantum whole are realized whereas others are not on the level of noetical apprehension and this is what can, in principle, be asserted for any particular contextual experimental frame.

From this aspect, the principle of actualization put up by R. Omnés as an additional external rule not emerging from the internal structure of **QM** to postulate the passing from phenomena to facts and used "merely as a licence to use consistent logic to reason from present brute experience" ([8], p. 1335) leads at least indirectly to an effective participancy of a knowing/intentional subject.

In phenomenological approach as the knowing/intentional subject acting as a constituting factor transposes the pre-phenomenological 'unity' of fundamental experience to the a-thematic, impredicative unity of its self-constituted flux of consciousness ([13], pp. 283, 295) it looks as if this underlying impredicative spatio-temporality should bear its 'imprint' on the interpretation of the wholeness of a quantum non-separable state standing as an undissectable whole and a limit to a complete scientific cognizance of physical reality. A prime reason for this limitation may lie in the fact that we lack any possible way to 'go deeper' than intentionality and consequently the analytical means to fully describe the inner time of an entangled state before or exactly at the stage of noetical apprehension; more generally it seems that we lack the means to unconditionally approach the temporality of the pre-phenomenological World (the World before the phenomenological reduction of the constituting *Ich*) which in Husserl's writings is presupposed as the constant synthetic unity of every possible experience and also the common denominator in terms of substance of all beings in the World. This phenomenological 'incompleteness' might somehow account for the inherent impossibility to provide a complete description of the World as a whole by means of a formal and logically consistent theory that would also include its universe (including the knowing/intentional subject) as its own object. Just as any language of an axiomatical system of mathematics capable of expressing at least elementary arithmetic cannot but eventually engender antinomies or paradoxes (cf. with Gödel's First Incompleteness Theorem) especially in relation to self-referential descriptions. In quantum world we could claim that this runs parallel to the example of von Neumann's account of quantum measurement that leads to an infinite regressional sequence of observing observers ([18], p. 306).

In this connection, I refer to M.L. dalla Chiara's view of the measurement problem of quantum mechanics as a characteristic question of the semantical closure of a theory, in other words as to "what extent a consistent theory (in this case **QM**) can be closed with respect to the objects and the concepts which are described and expressed in its metatheory". According to dalla Chiara, quantum mechanical theory as a consistent theory satisfying some standard formal requirements, turns out to be the subject of some limitations due to purely logical reasons concerning its capacity to completely describe and express certain physical objects and concepts.

Nevertheless, even if a contradiction produced in the metatheory of **QM** can be overcome on the purely logical grounds (linked to similar limitative results on the consistency of

axiomatical systems in set theory) that "any apparatus which realizes the reduction of the wave function is necessarily only a metatheoretical object" ([5], p. 338) the question, in my view, remains open of providing a consistent and complete metatheoretical description as to what 'happens' in physical state terms in-between the experimental preparation of a compound system such as $s \otimes \mathcal{Q}$ and the time of measurement corresponding to the collapse of the wave function (where s is a physical state at time t and $\mathcal{Q}$ a measuring apparatus identified with a Boolean-minded observer assigning truth values to non-Boolean quantum substructures). The jump of truth values in the process of measurement which is formally the result of the absence of an isomorphism between Boolean and non-Boolean structures - assuming that a quantum object, considered as an objective existence, is the non-distributive lattice of its properties - forces for a Boolean observer the need of the existence of an objective time in which he must 'move' ([10], p. 2396).

This question is also linked with J. von Neumann's Projection Postulate (or 'The Reduction of the Wave Function' postulate) as it implicitly establishes the necessity of a self-constituting time flux by assigning to the mathematical translation $\tau((s)(t))$ of a physical state $s(t)$ at time t the same eigenvector ψ_κ as for the measured quantity Q_i of the state $s(t)$ at time t_1 soon after the measurement.[22] As a matter of fact, even if we assume Von Neumann's Projection Postulate or Van Fraassen's modal interpretation of Quantum Mechanics as 'external' metatheoretical conditions in a purely logical way we cannot be led by any linguistic means to a complete description of the 'physical change' that takes place during the measurement process in the compound system 'system + apparatus'. This raises again the question of a self-constituting time flux of consciousness and the constitution of objects in it as noematical correlates of hyletical-noetical moments of an outward directed intentionality.

Closing the section I turn again to the fundamental irreducibilities which in my view shape in an essential way our observational frame in an intersubjective world of an unbounded horizon of events: On the one hand intentionalities of an *a priori* character directed on the lowest level of primordial experience to individuals - as such transposed then with their noematical correlates as immanences of the flux of each subject's consciousness. On the other, the intuition of continuous unity as a substratum divested of any quality on which to constitute and deliver a meaning to well-defined noematical objects described deeply enough as an impredicative self-constituting unity of the flux of consciousness leading ultimately to a transcendental ego of consciousness (see: [14], pp. 97-99).

Saying it in more intuitive terms, as much it is impossible to reduce the mental process by which we may abstract from an original impression in immediate awareness evidently distinct from any other in temporal flow to anything more fundamental in noetical apprehension it is equally impossible to capture what is constituted as the unity of a whole in consciousness by means of the former activity.

[22] By this postulate we get as a result of measurement the interval $r_\kappa \pm \epsilon_{Q_i}$, where r_κ is an eigenvalue of the mathematical form of Q_i and ψ_κ its corresponding eigenvector ([5], p. 344).

4 Observation in the language of formal systems. Where is the irreducibility and where the transcendence?

As the main purpose of this article was to discuss the limits of formal languages with respect to the notion of observation I naturally sought to reach the most fundamental level of observation beyond the limits of the common intuition of this notion. In doing so, I took also into account the claim, which was E. Husserl's belief, that mathematical objects are special cases of perceptual objects leaving aside any counter-arguments which are nevertheless of a rather artificial nature, e.g. whether the mathematical object $\{\emptyset\}$ should be also considered a perceptual object. My theoretical standpoint, linking fundamental observation to a phenomenology of constitution put under the same perspective the mental process of formation of mathematical objects as syntactical atoms corresponding to 'state-of-things' in a formal-ontological environment and the process of constitution of quantum entities as well defined noematical objects in consciousness based on their former intentional apprehension in the physical world.[23] It is obvious that in such an approach we should regard mathematics as divested of any platonist content and in a certain sense devoid of the conveniences of Cantorian-type actual infinity. In this connection, mathematical theories of an alternative nonstandard character especially those which incorporate e.g. a notion of natural infinity as an open-ended shift of classes of hereditarily finite 'observations' (for instance, Alternative Set Theory and Hyperfinite Set Theory) seem more adapted to my view of mathematical activity as a special kind of abstraction in an intersubjective and interactive field of events of a 'local' but ever receding horizon.

This way, talking of an object as an individual-as such it is in a fundamental and essential way the same whether it is a syntactical atom of a stratified mathematical formula corresponding to a unique 'state-of-things' or whether it is a quantum entity of a certain non-separable state apprehended by a disentangled interaction and transformed as a reidentifying-noematical object by a knowing/intentional subject using a measuring apparatus as an extension of his consciousness. As long as they can be apprehended as distinct to any other possible intentional apprehensions in the process of constitution they can both be classified as individuals and if in addition they can be constituted as bearers of an otherwise undefined sense of 'order' to any other such apprehension they can be classified as individual-substrates bearers by essence of an appropriate to them predicative environment (*Kernform*). Is there a way to penetrate more deeply, to open and 'read' the inner content of those individuals, in a word, to reach a deeper level of apprehension? The answer seems to be negative and in addition not in the sense of a contingent state of affairs but of a generic state of affairs. The main indication is our own intuition in the direct givenness of the intentional objects of our experience and this is why the intentionality of primordial

[23] My approach in this paper is not meant to be a transcendental deduction of Quantum Mechanics by means of Phenomenology for, notwithstanding my claim to providing some clues on a possible phenomenological interpretation of **QM** on the level of 'observation' and in metatheory, there are certain constants (the Planck constant, for instance) or symmetry principles of **QM** which are still doubtful whether they are of a purely empirical objective character or possibly susceptible of a phenomenological interpretation. Nevertheless, for the Planck constant, for example, there is a view of it as a not purely extrinsic datum but as arising from the generic situation of mankind which in my opinion leads indirectly to a notion of intersubjective constitution in the Life-World.

experience is described exactly as intentionality to intentional objects, i.e. to individuals-as such. I would call it a most fundamental irreducibility in relation to human perception though it is at the same time rather 'friendly' and easy to co-operate with our other mental faculties. It is thanks to this fundamental intuition that we can comprehend and handle almost anything from sequences of natural numbers to the capacity to shape images of various distinct particle trajectories in a bubble chamber.

On the contrary, there is another irreducibility which though it is our most common intuition it proves most difficult to comprehend let alone describe by formal first-order linguistic means. This is the intuition of Continuum which includes everything from the common intuition of our existence as a continuous state of events, to the intuition of a curve on a piece of paper as a continuous set of black points, to the intuition of subatomic events as taking place against a time-fulfilled continuous background. What is it that makes possible this coherent unity in constitution which is reflected as a formalized continuous substratum of the mathematical metatheory of 'observations' irrespective of whether they refer to a quantum mechanical context or to a context of common physical intuition?

Husserl made a clear distinction between phenomenological time, the homogenous form of all living experiences in the flux of consciousness and the objective or scientific time. By necessity every real experience is a durating experience which is a fact extracted by pure intuition of its enactment and it is constituted by a certain *a priori* mode as an infinitely fulfilled continuum of durations. Going deeper into the ontology of phenomenological Continuum Husserl encountered grave difficulties in comprehending it as he himself professed in *Ding und Raum* and he left it even in his later writings as a rather obscure notion leading him to a transcendental pure ego of consciousness. This ego as an absolute and impredicative subjectivity is only accessible by its objectivation as a 'mirror image' of itself and it is only in this way possible to reflect on the Continuum as an objective whole and also on the notion of an unbounded infinity in a Kantian sense ([12], p. 331). Essentially Husserl eradicated the transcendence of the world of the purest idealist doctrine only to introduce it backdoor by means of a personified pure ego.

The matter, in last count, is not so much whether one should in principle accept a transcendence of intuitive and consequently of mathematical Continuum by means of a phenomenological reduction or by some other interpretational scheme as the hard fact that any attempt to describe Continuum by the first-order linguistic means of a formal axiomatical system inevitably leads to circularities in definitions or entails some form of *ad hoc* axiomatization. This is presumably reflected in the independence of actual infinity principles such as *Continuum Hypothesis* and of the *Axiom of Choice* from the other axioms of the Zermelo-Fraenkel Set Theory and to some extent in the *ad hoc* extension or prolongation principles axiomatizing the embedding of standard structures into saturated nonstandard domains (see: [21]).

There is an ongoing theoretical discussion on the possibility of a non-analytical character of the *Continuum Hypothesis* question in the foundation of mathematics and on this account we refer to S. Feferman's thesis in [9] that "the *Continuum Hypothesis* is an inherently vague problem that no new axiom will settle in a convincingly definite way".

It seems worthwhile to close by mentioning a recent neuroscientific approach to the constitution of unitary and coherent experiences out of independent bits of data in which

the processes of predication and identity between different occurrences of variables (viewed as individuals) are reflections of the same underlying conceptual process in the brain. In [23] this process is described as diagrammatic and iconic rather than symbolic, in a certain sense as a neuronal continuous connection between differently localized predicates which involves also *Gestaltpsychological* notions substituting traditional symbolic operations.

But invoking some sort of continuous or topological representation in the context of neuronal processes described by first order formalism one might get trapped once more in the inherent impredicativity of the notion of Continuum. It is like swinging to the other end of the pendulum as we mark the following irreducibilities its extreme points; the intentionality to individuals-as such on the one hand and the intuition of the impredicative Continuum on the other.

The question of the nature of Continuum as well as the epistemic content of the aprioric principle of intentionality might be the object of a yet deeper research linking such diverse fields as mathematics, logic, quantum theory including its offspring quantum gravity, cognition theory and neurophysiology of the brain.

Or they might in principle be elusive and beyond any reach on the grounds of the circular question: On what terms can mind capture the mind?

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